Liquid Sloshing in Spherical Tanks

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This paper presents experimental data on the forced vibration characteristics of unbaffled and baffled spherical tanks partially filled with liquid. For the unbaffled tank, comparisons are made with previously measured natural frequencies and with the predictions of the Budiansky theory. For the baffled tanks, comparisons are made with the force response predicted from an equivalent mechanical model.

Introduction

THE design of large liquid propellant rockets and booster systems involves consideration of various tank configurations and the forces and moments resulting from sloshing of the contained liquids. In view of the customary high fineness ratio of such vehicles, most emphasis has been placed on cylindrical tanks;[1,2] however, in many instances where high strength-weight ratios are of great importance, in combination with other factors, spherical tanks have been employed. Knowledge of sloshing behavior in spherical tanks is considerably more limited than for cylindrical tanks, partially because of the increased complexity of theoretical analysis.

Virtually the only theoretical analysis available for spherical tank sloshing is that of Budiansky,[3] and, although somewhat sophisticated integral equation techniques were employed, the analysis is strictly valid only for the cases of the nearly empty, half full, and nearly full tank.§ The lowest three or four natural frequencies of the liquid free surface motion, over the entire range of liquid depths, have been determined experimentally by McCarty and Stephens,[5] confirming the theoretical values predicted by Budiansky[3] for the nearly empty and half full conditions.

The purpose of the present paper is to present experimental data on the forced vibration characteristics of partially filled unbaffled and baffled spherical tanks. For the unbaffled tank, comparisons are made with previously measured natural frequencies[5] and with the predictions of the Budiansky analysis.[3] For the baffled tanks, comparisons are made with the force response predicted from an equivalent mechanical model.‖

Experimental Results

Experimental Facilities and Procedures

The Southwest Research Institute slosh test facility, previously described in Ref. 7, was modified to accommodate a spherical tank approximately 15 in. in diameter (internal diameter of 14.5 in.). The tank was fabricated in two halves from 1020 steel sheet by spinning and was assembled by means of an equatorial flange; the tank was intended to be rigid. Instrumentation consisted of a dynamometer system for measurement of total force and moment response[7] and a small array of pressure cells. The excitation was translational in the horizontal direction.

For the studies of the effects of baffling, six perforated ring baffles of 23% open area were employed. The baffles were fabricated from 0.016-in.-thick perforated brass sheet having 0.020-in.-diam holes and were mounted in the tank so as to form great circles at 30° to each other. The baffle width was 2.06 in. (baffle width to radius ratio of 28.5%). Tests were conducted with the baffles in two different orientations; the horizontal orientation is shown in Fig. 1.# The vertical orientation was formed simply by rotating the sphere 90°, so as to form lines of longitude. In both orientations, the same cross section of the sphere (containing one of the baffles) is normal to the translational excitation axis; hence, in Fig. 1 the axis of translation is normal to the page.

Total Force Response: Unbaffled Tank

As a matter of simple comparison, previous data[3,5] for first mode natural liquid frequency variation with depth are shown, together with present Southwest Research Institute data, in Fig. 2. The natural frequency ω_n is given in terms of the nondimensional parameter $\omega_n^2 d/a$, where a represents a vertical acceleration field and d is the tank diameter.

Total force response, in terms of dimensionless amplitude and phase angle, is shown as a function of the frequency parameter at various liquid depths in Figs. 3–6. The vertical line in each plot represents the fundamental liquid resonant frequency.

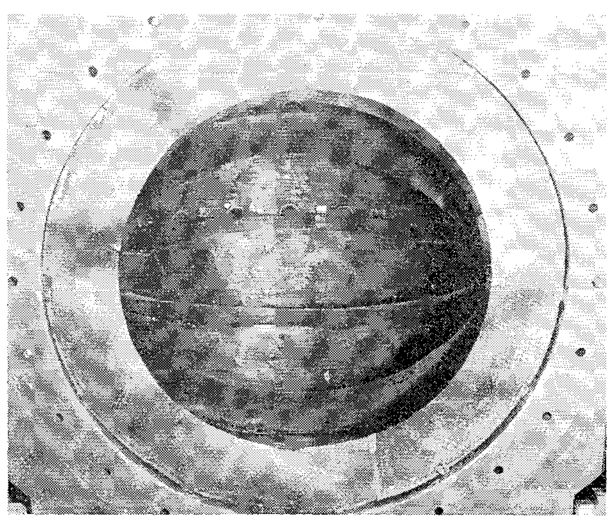

Fig. 1 Horizontal baffle arrangement

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§ The results of a generalized computer program have been given in Ref. 4 and found to agree well with those of Budiansky.

‖ This analogy was developed by W. H. Chu and is given in Ref. 6.

Certain of the pressure cells also may be seen in Fig. 1.

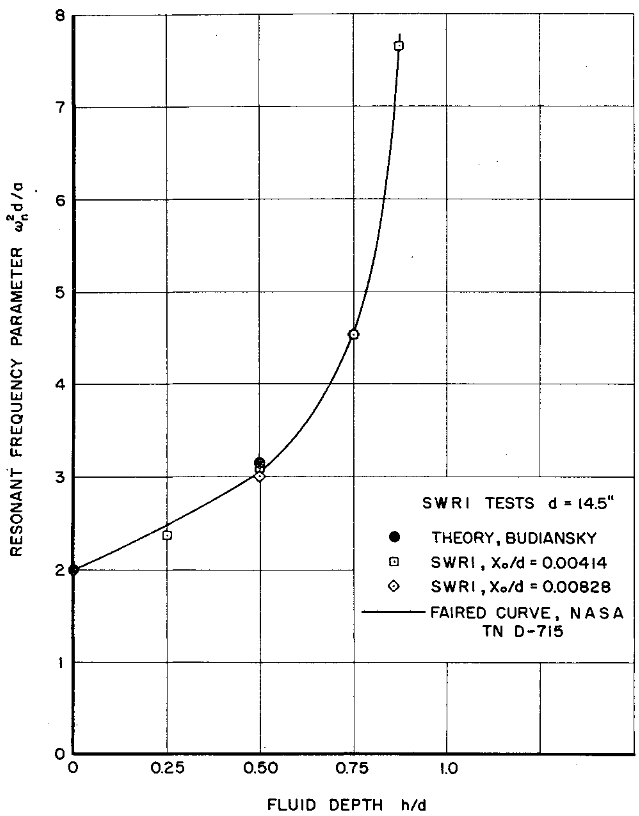

Fig. 2 Liquid fundamental resonant frequency variation with depth

The effect of different excitation amplitudes X_0 is shown in Figs. 4–6. Lower values of excitation amplitude generally appear to produce somewhat greater response for frequencies less than the fundamental, and lower response for frequencies greater than the fundamental. Figure 6, corresponding to a nearly full tank, shows a prominent peak in response only for the lower excitation amplitude, the higher excitation amplitude apparently introducing sufficiently large amplitudes of free surface motions that the damping becomes quite high.

Figures 3–5 also contain theoretical force response curves as obtained from Budiansky's analysis.[3]** The in-phase portions of these curves generally show good agreement between theory and experiment; the out-of-phase portions show somewhat poorer agreement, with the experimental data corresponding to lower excitation amplitudes being much closer to the theoretical curves than experimental data corresponding to the higher excitation amplitude. The effects of large excitation amplitudes therefore are emphasized again.

As a further comparison between the Budiansky theory[3] and the experimental response data, experimentally determined pressure distribution data for $h/d = 0.50$ and $X_0/d = 0.00828$ (high excitation amplitude) were integrated to give total force, as shown in Fig. 4. The agreement with the Budiansky theory is good; this, however, is probably only a fortuitous circumstance because the pressure data are not sufficiently detailed to permit accurate integration (see below).

Pressure Distribution: Unbaffled Tank

Figure 7 shows experimental wall pressure distribution data similar to those presented previously for cylindrical tanks.[8] The wall pressures are given in nondimensional form as a function of pressure cell location z/d (measured vertically from the tank bottom), with excitation frequency as a parameter. The pressures are, of course, those normal to the tank wall. A few measured values also were obtained along lines of latitude at $z/d = 0.5$, 0.375, and 0.25. As is evident from Fig. 7, there are not sufficient measured values to insure accurate pressure distribution curves, particularly as required for integration to obtain total force response (see the foregoing); the data given therefore are useful primarily to indicate trends and magnitudes. The magnitudes of the pres-

** Since the theory is strictly valid only for $h/d \simeq 0$ and 0.50, the values for $h/d = 0.25$ and 0.75 were obtained by estimating values from the interpolated curves given in Ref. 3, with consequent inaccuracies.

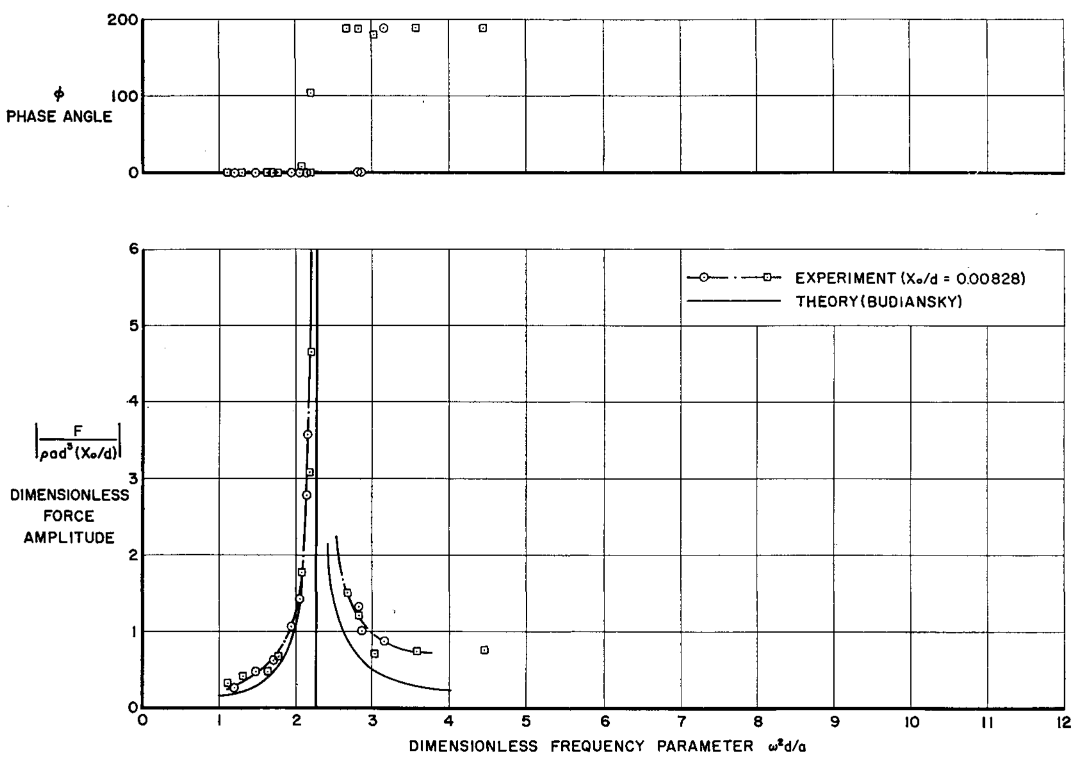

Fig. 3 Liquid total force response, unbaffled tank ($h/d = 0.250$)

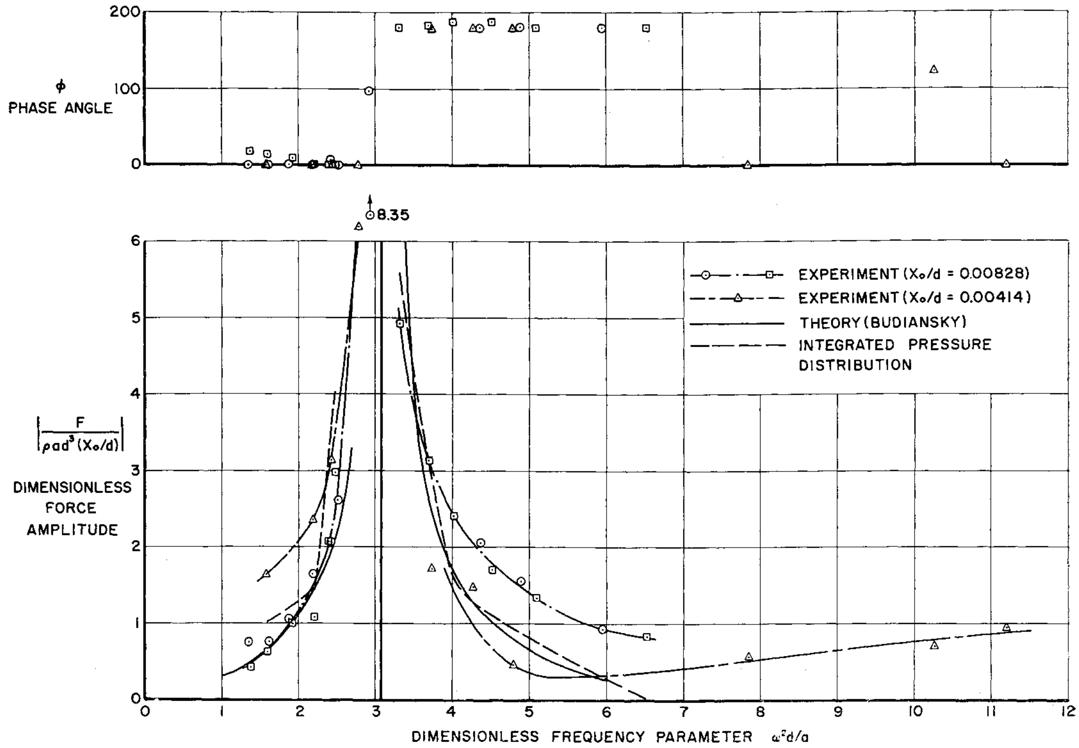

Fig. 4 Liquid total force response, unbaffled tank (h/d = 0.500)

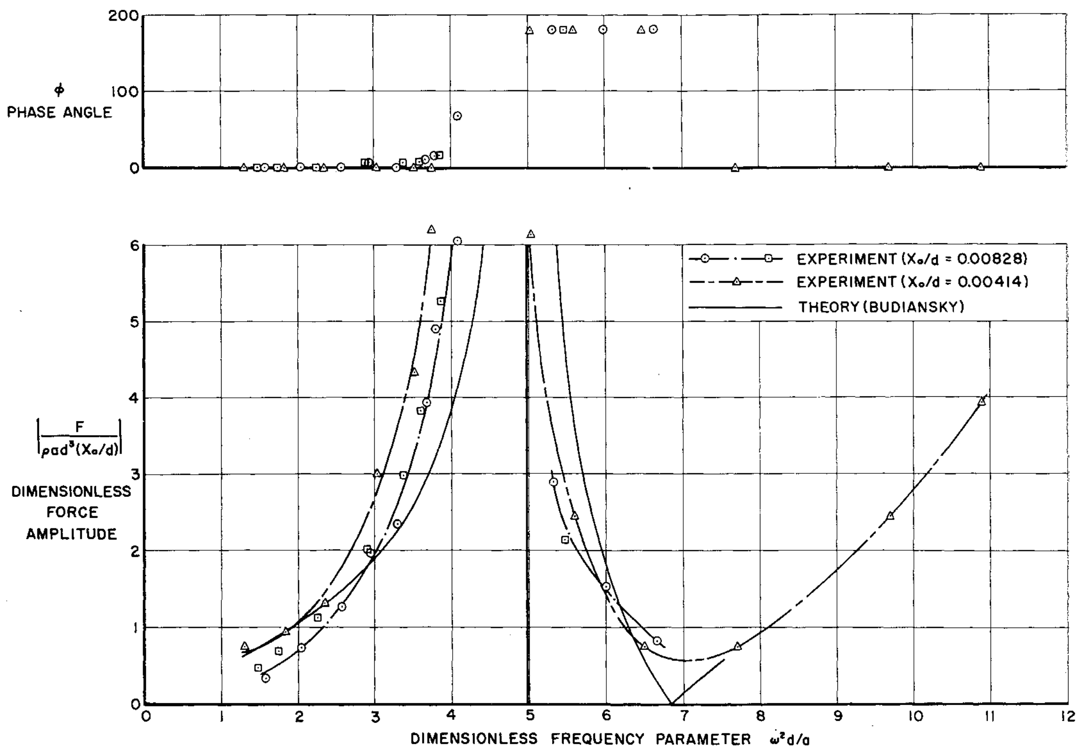

Fig. 5 Liquid total force response, unbaffled tank (h/d = 0.750)

sures are, incidentally, considerably higher than for an equivalent cylindrical tank.[8]

Total Force Response: Baffled Tank

Total force response data for the spherical tank with either horizontal or vertical perforated ring baffles (Fig. 1) are shown in Figs. 8 and 9. Both baffle orientations provide large force amplitude damping, with the horizontal arrangement being considerably more effective in this respect than the vertical arrangement. This is expected from the orientation of the rings with respect to the liquid free surface.

For rather full unbaffled tanks ($h/d > 0.75$), as may be inferred from Fig. 6, the peak force response is sufficiently small so as to be more like that of a well-baffled tank. In fact, for $h/d = 0.875$ and $X_0/d = 0.00828$, the force response for the unbaffled tank and for the tank with either horizontal or vertical baffles are virtually the same. Hence, one could conclude that baffling is required in spherical tanks only for

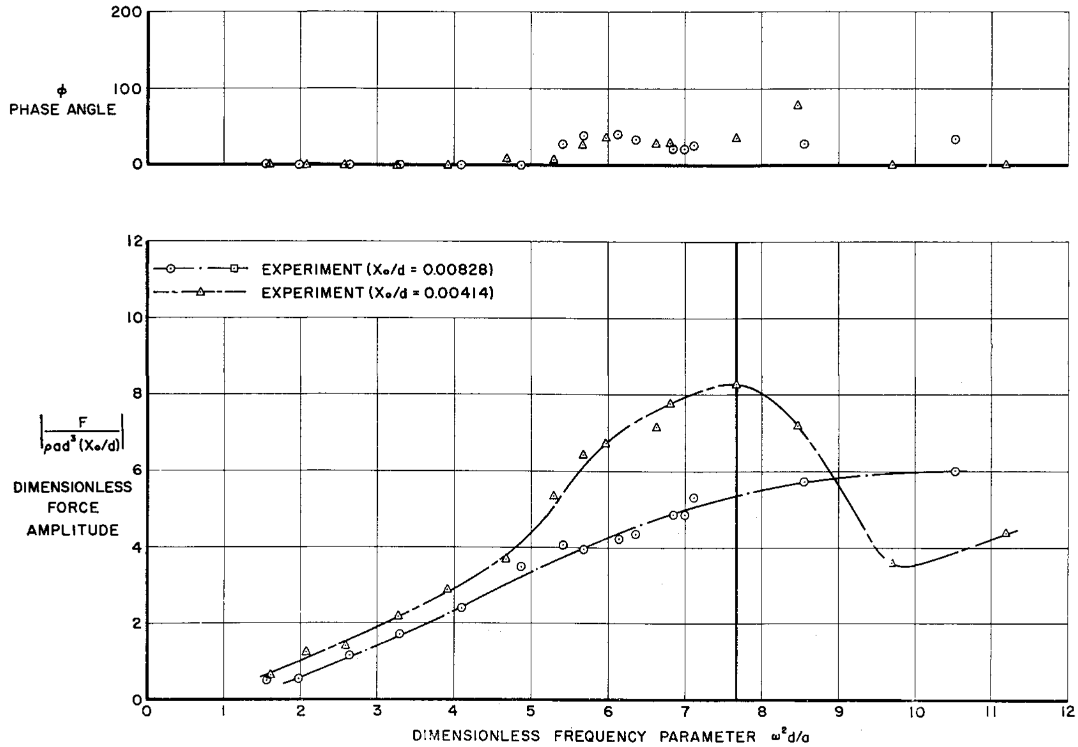

Fig. 6 Liquid total force response, unbaffled tank (h/d = 0.875)

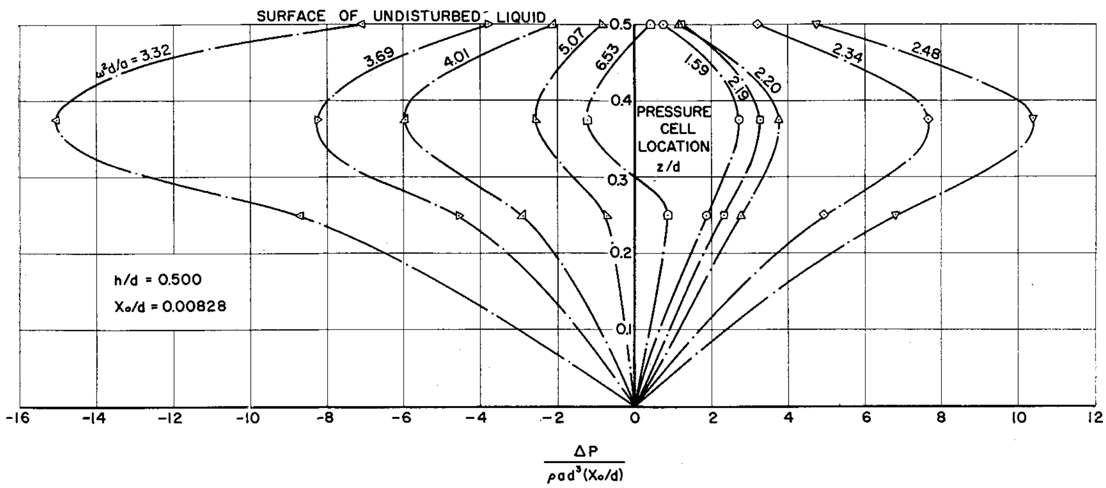

Fig. 7 Liquid wall pressure distribution, unbaffled tank

$h/d \leq 0.75$ at the excitation amplitudes considered. For very shallow liquid depths, say h/d = 0.25, the horizontal baffle orientation provides almost complete suppression of the force response. As may be noted particularly from both Figs. 8 and 9, the presence of baffles also has a significant effect in lowering the fundamental liquid resonant frequency.

In order to place these data on baffle effectiveness on a somewhat more quantitative basis, an attempt was made to obtain values of damping factor g. The half-bandwidth procedure applied to the experimental force response peaks, as employed in previous work,[7] could give g values in only a few instances, however, because of the rather large damping provided by both of these baffle configurations. As noted previously, for large depths (such as h/d = 0.875) the unbaffled tank response is so highly damped in itself that baffling has no additional effect; for shallow depths (such as h/d = 0.25) the damping provided by either baffle configuration is also so large as to suppress completely the liquid free surface motion. Even for the half-full case of h/d = 0.50, the horizontal baffle configuration is so effective as virtually to eliminate the response peak (Fig. 8); at h/d = 0.75 (X_0/d = 0.00828) the horizontal baffle yields a damping factor of g = 0.65, which is a quite large value (Fig. 9). The vertical baffle configuration yields a damping factor g = 0.31 for depths of h/d = 0.50 and 0.75 for an excitation amplitude of X_0/d = 0.00828 (Figs. 8 and 9). It is emphasized that these g values are given here primarily for demonstrating the orders of magnitude involved and for comparison with similar data for cylindrical tanks.[7]

Equivalent Mechanical Model

In order to be able to incorporate liquid force response data into vehicle dynamic analyses, recourse is often made to representation by an equivalent mechanical model. By incorporating viscous dashpots into the mechanical elements of the model, it therefore is possible to employ such models for representation of baffled tanks.††

†† See Ref. 9 for a dicussion of mechanical models for cylindrical tanks. The derivation of the mechanical model for the present case of a spherical tank is given in Ref. 6.

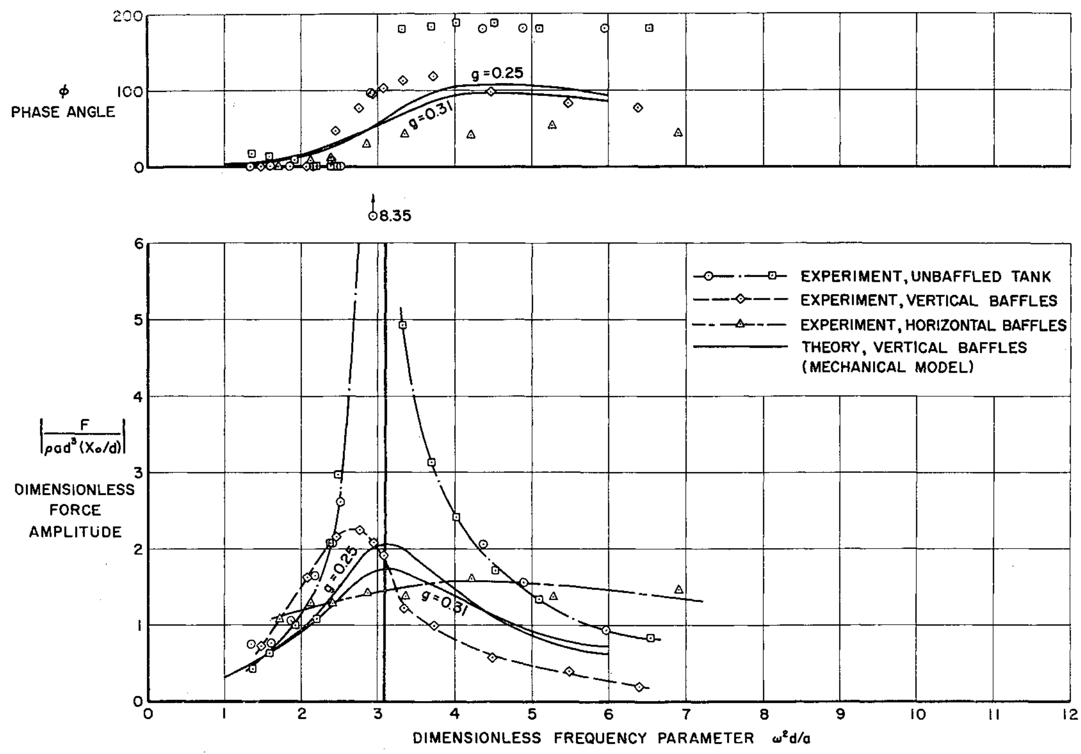

Fig. 8 **Liquid total force response, baffled tank (h/d = 0.500, X_0/d = 0.00828)**

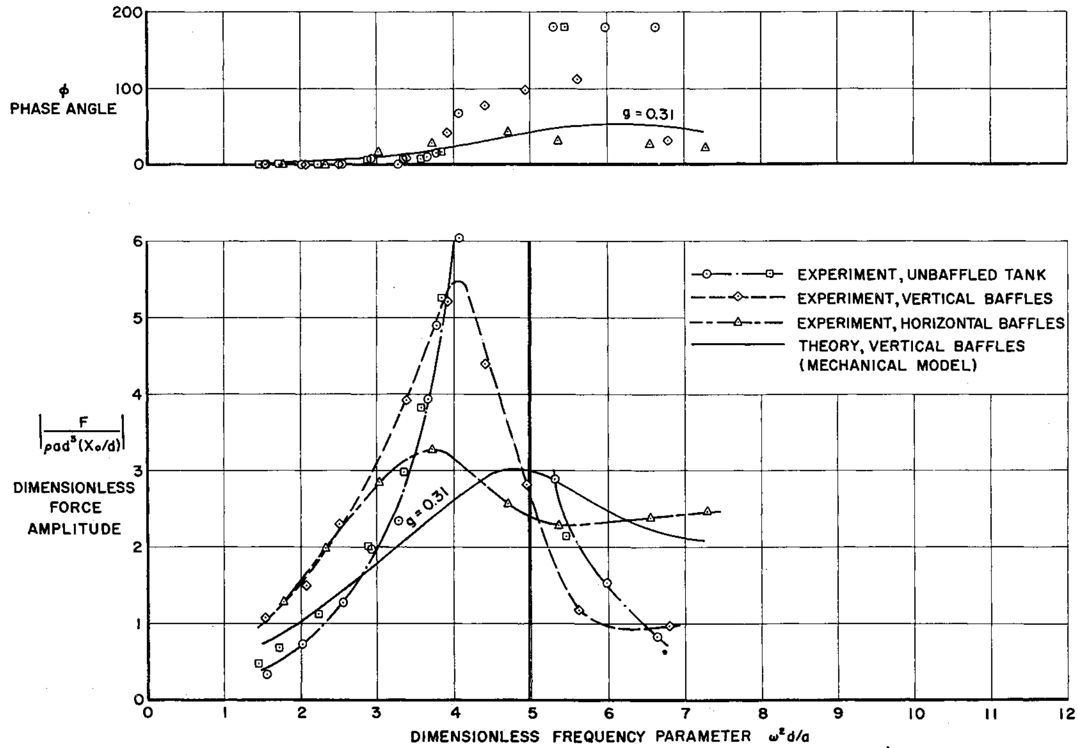

Fig. 9 **Liquid total force response, baffled tank (h/d = 0.750, X_0/d = 0.00828)**

Using the g values given in the preceding section, force response curves have been calculated from the mechanical model for the spherical tank with vertical baffles with $h/d = 0.50$ and 0.75, as shown in Figs. 8 and 9. In both cases, the predicted values of the peak response are considerably below the measured values, whereas the value of the frequency parameter at which the peak occurs is considerably higher for the calculated data than for the measured data. Recognizing that the g values obtained from the experimental data probably are not too accurate, the case of vertical baffles at $h/d = 0.50$ also was calculated for a smaller value of damping ($g = 0.25$) rather than the value of $g = 0.31$ originally obtained, with consequent better agreement in peak value with the measured data (see Fig. 8).

As has been noted in the earlier discussion of this paper, sloshing in spherical tanks appears to involve much larger free surface amplitudes than is customarily observed in cylindrical tanks, even at very low excitation amplitudes, so that an essentially larger degree of nonlinearity is present. This is particularly evidenced by the differences between the measured force response and that predicted by theory and the very large damping and reduction in liquid natural frequencies produced by both baffle arrangements. Thus, although the response curves calculated from the equivalent mechanical model representation, based on damping values obtained from the measured data, are not in particularly good agreement with the measured force response, the disagreement is probably no more than should be expected from a linearized representation.

Discussion and Conclusions

The results of this study would appear to reveal several important features of sloshing in spherical tanks. For example, although the basic predictions of the Budiansky theory[3] have been confirmed, particularly for liquid natural frequency, the force response is quite dependent on the magnitude of the excitation amplitude. Hence, large amplitude liquid free surface motions are excited more easily and appear to be of more importance in modifying the total force response in spherical tanks than in cylindrical tanks.

Perforated ring baffles oriented horizontally appear to be quite effective in providing force amplitude damping, with significant lowering of the fundamental resonant frequency. No baffling apparently is required for large liquid depths, say $h/d > 0.75$. A linear mechanical model representation for baffled tanks would appear to be satisfactory only for order of magnitude estimates.

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Safety Information from Propellant Sensitivity Studies

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This paper summarizes recent results of propellant sensitivity studies at the Naval Ordnance Laboratory. It emphasizes the use of gap and other small scale test results to help answer practical safety questions. Measured initiating pressures are highest near the critical diameter of the test material and decrease to their lowest value, for a given donor, at an effectively infinite acceptor diameter. The confinement of the standardized gap test increases the effective acceptor by about 2.5 times for Composition B. Gap test results show good correlation with large-scaled field tests and the approximately one-dimensional wedge test results; it is therefore believed that the gap test measures a 50% initiating pressure very close to that for an infinite diameter acceptor. A supplement to the Naval Ordnance Laboratory shock sensitivity test for propellants has been devised whereby the judicious choice of explosive witness systems makes it possible: 1) to assess the strength of reactions of too low impulse to produce a positive result under the conditions of the standardized test, and 2) to measure the sensitivity to shock initiation of substances exhibiting such reactions.

THE objective of the continuing work on propellant sensitivity is to understand the process of initiation (by any external stimulus) and any subsequent self-propagating reaction. With sufficiently detailed information, it should be possible to answer practical questions arising from safety considerations. Some of the important questions are as follows:

1 How easily will ignition and propagation of burning occur?

2 Can detonation occur?

3 If so, what is the probability of transition from burning to detonation?

4 What damage will a runaway reaction cause?

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It has been found (1)[3] that propellants generally exhibit high to very high impact sensitivity. Hence, these substances are easy to ignite and burn, a characteristic to be expected of materials used as propellants. Work is underway at many laboratories to assess the degree of fire hazard, and much still remains to be done in this field, but the present work has been concentrated on reactions more damaging than simple combustion. Any easily combustible material capable of energetic exothermal reaction immediately suggests the possibility of detonation. By use of a standardized gap test (1, 2), it has been possible to obtain much information on whether a propellant is detonable and, if so, how easily. Since the gap or shock sensitivity test measures the minimum initiating pressure required to induce detonation, it also provides some information about the probability of an occur-

[3] Numbers in parentheses indicate References at end of paper.